Derivation of the 3 Formulas for constant acceleration

$\bar{a} = \frac{\Delta v}{\Delta t} \rightarrow a = \frac{v_f - v_i}{\Delta t}$ (If acceleration is constant then $\bar{a} = a$)

multiplying both sides by Δt, $a\Delta t = v_f - v_i$

Rearranging, $\boxed{v_f = v_i + a\Delta t}$

Equation 1

$\bar{v} = \frac{\Delta x}{\Delta t} \rightarrow \Delta x = \bar{v}\Delta t$

replacing $\bar{v}$ with $\frac{v_f + v_i}{2}$, $\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t$ *

replacing v_f with equation 1, $\Delta x = \left(\frac{v_i + v_i + a\Delta t}{2}\right)\Delta t$

distributing, $\Delta x = \frac{2v_i\Delta t}{2} + \frac{a\Delta t^2}{2}$

simplifying, $\boxed{\Delta x = v_i\Delta t + \frac{1}{2}a\Delta t^2}$

Equation 2

Starting with *

$\Delta x = \left(\frac{v_i + v_f}{2}\right)\Delta t$

Replace Δt,

$v_f = v_i + a\Delta t$

$\Delta t = \frac{v_f - v_i}{a}$

$$\Delta x = \left(\frac{v_i + v_F}{2}\right)\left(\frac{v_F - v_i}{a}\right)$$

Multiplying both sides by $2a$,

$$2a\Delta x = (v_i + v_F)(v_F - v_i)$$

Foiling and crossing off 2 terms that cancel

$$2a\Delta x = \cancel{v_i v_F} - v_i^2 + v_F^2 - \cancel{v_F v_i}$$

$$2a\Delta x = -v_i^2 + v_F^2$$

Rearranging,

$$\boxed{v_F^2 = v_i^2 + 2a\Delta x}$$

Equation 3

Now we have 3 more equations we can use when studying motion. All are valid if acceleration is constant. Sometimes you will have a choice of which equation to use. Other times there will only be one equation that you can use based on what information is known.

You will not be asked to derive any of these equations. I only include the derivations to show you that these new formulas come from manipulating the old formulas we already knew. We don't need these new formulas to solve motion problems, but they can make the math easier at times. For example, if given a, v_i, and Δt we can solve for v_F by using $\bar{a} = \frac{\Delta v}{\Delta t}$, but it's much easier to solve using equation 1 instead ($v_F = v_i + a\Delta t$).

Practice with the 3 Formulas

① $V_f = V_i + a\Delta t$

② $\Delta x = V_i \Delta t + \frac{1}{2} a \Delta t^2$

③ $V_f^2 = V_i^2 + 2a\Delta x$

V_f = final velocity (m/s)
V_i = initial velocity (m/s)
a = acceleration (m/s^2)
Δt = time (s)
Δx = displacement (m)

For each problem, write your given information, then choose which formula to use based on what you know and what you are trying to solve.

Example ①

A truck starts from rest and accelerates uniformly at $2\ m/s^2$ for 8 seconds. How far does it travel?

Example ②

A car traveling at $30\ m/s$ slams on its brakes and slows down at a rate of $5\ m/s^2$. How far does it travel before stopping?

Example ③

A person walking in a dark alley accelerates at $0.8\ m/s^2$ for 4 seconds after hearing a noise behind him. If his final velocity is 5.2 m/s, what was his initial walking speed?

Example ④

A ball starts from rest and rolls down a 3 meter long ramp and attains a final velocity of 4.2 m/s.

A) What is the ball's average velocity?

B) What is the ball's acceleration down the ramp?

C) How long does it take the ball to roll down the ramp?

Answers

1. 64 m
2. 90 m
3. 2 m/s
4. 2.1 m/s, $2.9\ m/s^2$, 1.4 s